Baroclinic dynamics of wind-driven circulation in a stratified bay: A numerical study using models of varying complexity

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ABSTRACT

The baroclinic response of a stratified coastal embayment (Lunenburg Bay of Nova Scotia) to the observed wind forcing is examined using two numerical models. A linear baroclinic model based on the normal mode approach shows skill at reproducing the observed isotherm movements and sub-surface currents during a time of strong stratification in the bay. The linear model also shows that the isotherm movement in Lunenburg Bay is influenced by the wind forcing and propagation of baroclinic Kelvin waves from neighbouring Mahone Bay. The effects of nonlinearity and topography are investigated using a three-dimensional nonlinear coastal circulation model. The nonlinear model results demonstrate that the nonlinear advection terms generate a gyre circulation at the entrance of Lunenburg Bay, and the slope bottom topography at the mouth of the bay strengthens the sub-surface time-mean inflow on the southern side of the bay. A comparison of model-calculated currents in different numerical experiments clearly shows that baroclinicity plays a dominant role in the dynamics of wind-driven circulation in Lunenburg Bay.

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1. Introduction

Circulation over eastern Canadian coastal waters is affected by tides, wind and buoyancy forcing associated with heat and fresh water fluxes. The highly irregular coastline and variable bottom topography in the region pose a great challenge that has to be overcome if we are to understand and predict the hydrography and three-dimensional circulation of the region. Lunenburg Bay (LB) of Nova Scotia, Canada (Fig. 1) was therefore chosen as a test site for the development of a prototype modeling and observation network that could be used for marine prediction in the region. Better understanding of the dynamic response of coastal waters to forcing will help improve our ability to accurately model coastal circulation and hydrography. The barotropic dynamics of LB have been extensively studied (Sturley and Bowen, 1996; Thompson et al., 1998; Sheng and Wang, 2004; Wang et al., 2007; Sheng et al., 2008; Mulligan et al., 2008), whereas our understanding of the baroclinic dynamics of LB is still in its infancy (a start has been made in Zhai et al., 2007, 2008). Wang et al. (2007) demonstrated that the barotropic circulation in the bay is significantly affected by local wind. Zhai et al. (2008) showed that the baroclinic circulation and associated high frequency variation (time scales of 1–10 days) of temperature and salinity in the bay are also mainly forced by local wind. The semi-diurnal (M2) tidal currents also play a role in affecting temperature and salinity distributions in the bay, particularly near Corkum’s Channel and in the two connecting coves. In this study, we try to better understand the baroclinic response of LB to wind forcing using a hierarchy of models ranging from a linear baroclinic model, based on the normal mode approach with a flat bottom, to a fully nonlinear model with variable bottom topography.

The baroclinic response of coastal bays and inlets to wind forcing is generally characterized by coastal upwelling/downwelling and baroclinic coastal trapped waves, and can sometimes be approximated by linear baroclinic dynamics. Three types of baroclinic coastal waves were studied in the past, including baroclinic Kelvin waves, baroclinic Poincaré waves and topographic waves. Csanady and Scott (1974) studied baroclinic coastal jets in Lake Ontario using a theoretical model for a two-layer, long and narrow lake. Wang (1975) examined the baroclinic Kelvin waves and topographic Rossby waves in a two-layer water with a slope bottom. Crépon and Richez (1982) and Crépon et al. (1984) investigated the effect of capes and bays on the coastal upwelling analytically, and showed that Kelvin waves and Poincaré waves can be generated by the irregular coastline. deYoung et al. (1993b) demonstrated the nonlocal effect of Kelvin wave propagation into Conception Bay from neighbouring Trinity Bay in Newfoundland Canada, and Davidson et al. (2001) further showed that the asymmetric response of Conception Bay to wind forcing is associated with the higher-order baroclinic modes. Gan et al.
main physical processes necessary to understand the baroclinic circulation and upwelling and downwelling in the bay. The main advantage of the normal mode approach is that the governing equations for the mode coefficients and vertical normal modes are separated and that the baroclinic response is easily illustrated (Gill, 1982). We then use the nonlinear coastal circulation model described by Zhai et al. (2008) to examine the nonlinear and topographic effects. The arrangement of the paper is as follows. In Section 2, the main features of the observations relevant to this study are briefly discussed together with some thoughts on the relevant dynamics. In Section 3, the multi-mode model is applied to LB and Mahone Bay (MB) with a flat bottom to investigate the remote effect of propagation of baroclinic Kelvin waves from MB to LB. In Section 4 the fully nonlinear and three-dimensional coastal circulation model described in Zhai et al. (2008) is used to investigate the influence of topography and nonlinearity neglected in the linear multi-mode model. Results are discussed, and conclusions are made in Section 5.

2. Observations and dynamical considerations in LB

Three moorings (shown in Fig. 1) were deployed at sites SB2, SB3 and MB1 in LB to acquire observations of wind vectors, hydrography and currents (see Zhai et al., 2007, for the details). August 13 (day 224) to September 7 (day 249) in 2003 is chosen as the study period. This was a period with strong vertical stratification in LB, during which the baroclinicity plays an important role in the coastal circulation in the bay. The wind stress calculated from the observed wind speed at the mooring SB3 as well as observed temperatures at different depths at SB3, SB2 and MB1 during this period are shown in Fig. 2. The bulk formula suggested by Large and Pond (1981) is used in converting the observed wind velocity to wind stress. Mean wind stress during this period is about 0.18 dyn cm⁻², and points in the direction of ~22° measured anti-clockwise from east. The standard deviation of the wind stress is greater than the mean with a magnitude of 0.42 dyn cm⁻², 68% of which is associated with the major axis (64.2° measured anti-clockwise from east and pointing northward across the bay), and the remaining variance is associated with the minor axis. Multiple regression analysis indicates that the variability of observed temperatures at the three mooring sites (shown in Fig. 1) can be partially explained by the past history (within 12 h) of wind forcing during the period (Zhai et al., 2007). The observed temperatures at fixed depths can be used to estimate the vertical motion of the isotherm. The decrease (increase) in temperatures at different depths at the three moorings is associated with the upward (downward) movement of the isotherm. From day 233 (August 21) to day 238 when the wind stress changed from northeastward (across-bay) to southeastward (along bay), the observed temperatures at the three mooring locations decreased first due to the upwelling induced by Ekman divergence, and then gradually increased due to downwelling (Fig. 2). We show in the next section that this downwelling event is mainly associated with the propagation of baroclinic Kelvin waves from MB to LB.

Previous studies demonstrated that the strong stratification in LB in August and early September of 2003 was mainly maintained by the heat and fresh water input at the sea surface and the wind-induced upwelling of the relatively cold and saline near-bottom waters (Zhai et al., 2007, 2008). Based on the area-averaged density profile (Fig. 3a) made by the CTD surveys on September 6, 2003, which is representative during the study period, the first four baroclinic vertical modes (Fig. 3c) are calculated from the buoyancy frequency \( N^2 = -g(\rho_0 / \rho)(\sigma / \partial z) \), where \( g = 9.8 \text{ m s}^{-2}, \rho_0 = 1024 \text{ kg m}^{-3} \) and \( \sigma \) is the in situ density (see Eq. (A.2) in

Fig. 1. Model domains of (a) Mahone Bay (MB), Lunenburg Bay (LB) and Rose Bay (RB), and (b) Lunenburg Bay and Rose Bay. Abbreviations used: Big Tancook Island (BTI), Little Tancook Island (LTI), Upper South Cove (USC), Lower South Cove (LSC), East Point Island (EPI), Cross Island (CI) and Corkum’s Channel (CC). The solid triangles in (b) denote mooring buoys. Model results along the transect due north from Ovens Point (OP) in (b) are discussed in Sections 3 and 4.
Appendix A, or Kundu, 1990). The phase speeds for the first four baroclinic modes \(c_n\) are 0.22, 0.11, 0.07 and 0.06 m/s, respectively. The dimension of LB is about 4 km × 8 km and is larger than the first baroclinic Rossby radius of 2.2 km, indicating that the rotation effect is important in the baroclinic dynamics in the bay. Previous studies demonstrated that both internal Kelvin waves and Poincaré waves can be generated in coastal waters (Crépon and Richez, 1982; Gill, 1982). The power spectra of observed non-tidal currents, temperatures and salinities in LB are dominated by the energy in the frequency band of less than the inertial frequency (17 h) at the three mooring locations (not shown). The \(M_2\) tidal circulation also plays an important role in generating hydrographic variability with frequencies of greater than the inertial frequency in the bay, particularly in the two coves and Corkum’s Channel. The detailed study of super-inertial waves (i.e. Poincaré waves) is beyond the scope of this study, and we focus only on the sub-inertial baroclinic Kelvin waves in this study.

The momentum balance analysis for the depth-averaged currents produced by the numerical model suggests a “modified” geostrophic balance, in which wind stress minus bottom stress and advection also play a role on the time scale of \(\approx 10\) days in LB (Zhai et al., 2008). The head of LB is relatively shallow of \(\approx 8\) m, and the maximum depth is about 25 m at the mouth (Fig. 1). There is a shoaling area to the northern side of the bay entrance. Based on the arrested topographic wave theory (Csanady, 1978), the characteristic distance that the sea level (barotropic pressure) in response to steady wind is trapped from the coast is given as

\[
L = \left( \frac{f dh}{r dy/L_x} \right)^{-1/2}
\]

(1)

where \(f\) is the Coriolis parameter and set to \(10^{-4}\) s\(^{-1}\) for LB, \(r\) is the bottom friction and set to \(10^{-3}\) m/s, \(dh/dy\) is the across-bay slope at the mouth of LB and set to \(2.5 \times 10^{-3}\), and \(L_x\) is the bay length. In LB, \(L\) has a value of 5 km and is comparable to the bay width, thus the local topography at the bay entrance is expected to have some influence on the low-frequency (>10 days) circulation in the bay. Previous studies (Furnes, 1980; deYoung et al., 1993a; Winant, 2004; Sanay and Valle-Levinson, 2005) have already shown the importance of topography on the barotropic wind-driven circulation in coastal sea regions.

3. The linear baroclinic dynamics

In this section, the baroclinic response of LB to wind forcing is examined using the linear dynamical model based on the normal mode approach for a continuously stratified ocean of uniform depth. The linear multi-mode model was successfully used to study coastal and equatorial upwelling in the past (Gill and Clarke, 1974; McCreary 1981a, b; deYoung et al., 1993b; Davidson et al., 2001). The application of the multi-mode model to LB is justified since

\[
S = N^2H^2/f^2 \sim 10 \gg 1, \quad \text{(Huthnance, 1978)}
\]

by setting \(H = 10\) m, \(N = 0.03\) s\(^{-1}\) and \(l = 1\) km for LB, indicative of the importance of baroclinic Kelvin waves. We consider small perturbations of currents from a state of rest and small perturbations of density from a specified horizontally uniform stratification taken as the September 6 profile shown in Fig. 3a. The dynamic equations (McCreary, 1981a; see Appendix A) are linearized about the undisturbed state and solved by expressing the state variables in terms of the vertical normal modes, each of
which has a fixed vertical structure (Fig. 3c). It should be noted that the normal mode approach requires an approximation of a flat ocean bottom, which is one of the limitations of the method.

The governing equations for the mode coefficients are the same as the shallow water equations in a homogeneous fluid with a free surface but with “depth” replaced by “equivalent depth” (Gill, 1982). In this study, the shallow water equations for the mode coefficients are solved numerically following Heaps (1971). The open boundary conditions are the same as those given by Greatbatch and Otterson (1991). A feature of their boundary conditions is the treatment at the upstream boundary (in the sense of Kelvin wave propagation), where spurious Kelvin waves entering the model domain are prevented. We follow McCreary (1981a) and parameterize the vertical eddy viscosity and diffusivity as $K_m = \sigma K_h = A/N^2$, with $\sigma = 0.1$ and $A = 2 \times 10^{-2} \text{cm}^2 \text{s}^{-1}$. Therefore, the e-folding damping time of each mode varies as $c_n^2$, where $c_n$ is the baroclinic wave speed for the $n$th mode (see Eq. (A.6) in Appendix A), which means that higher vertical mode waves, for which $c_n$ varies as $1/n$, are damped faster than lower modes. Here $A$ is chosen so that $K_m = 10 \text{cm}^2 \text{s}^{-1}$ when $N = 0.03 \text{s}^{-1}$ (the depth-mean value), indicating that the Ekman layer depth $\sqrt{2K_m/f}$ is $\sim 4.5 \text{m}$, which is generally consistent with the mixed layer depth estimated from the temperature measurements in LB during the study period. Ten baroclinic modes are used in the numerical calculation, which are sufficient to ensure convergence of the numerical results.

Two numerical experiments are conducted using the linear multi-mode model. The model domain uses the realistic geometry of LB (Fig. 1b) in the first experiment (Mode-LB) and uses the geometry of LB and MB (Fig. 1a) in the second experiment (Mode-MLB) (Table 1). In both the experiments, the model water depth is uniform and set to 15 m. The horizontal resolution in both experiments is about 200 m, which is sufficient to resolve the Rossby radii for the first three baroclinic modes. The vertical grid spacing used to solve for the vertical structure functions is 1 m.

The linear multi-mode model in both the experiments is forced by the time-varying but spatially uniform wind stress shown in Fig. 2a and integrated for 25 days from August 13 to September 7, 2003, with the initial state being one of rest and zero density anomaly everywhere. Here the density anomaly is defined as the difference between the total density and the background density profile. The latter is a function only of depth and is shown in

![Fig. 3. Vertical profiles of (a) in situ density $\sigma_t$, (b) buoyancy frequency and (c) pressure eigenfunctions of the first four baroclinic modes in Lunenburg Bay in September 2003. Phase speeds for the first four baroclinic modes are about 0.22, 0.11, 0.07 and 0.06 m s$^{-1}$, respectively. The units of $\sigma_t$ and $N$ are kg m$^{-3}$ and s$^{-1}$, respectively.](image-url)
Fig. 3a. Model results are described first to explain the propagation of baroclinic Kelvin wave, followed by the comparison with observations to quantify the feasibility and limitation of the linear multi-mode model.

### 3.1. Local effect of LB

The near-surface currents and density anomaly fields at three different times produced by the linear multi-mode model in experiment Mode-LB are shown in Fig. 4. The wind stress (Fig. 2) is about 0.1 Pa and southeastward at day 235.5 and gradually dies down during the two-day period. At day 235.5, the near-surface baroclinic currents (Fig. 4a) flow northeastward over western LB and turn eastward over eastern LB, indicative of the geostrophic balance. To the east of LB the near-surface flow is southward and nearly in Ekman balance. Positive density anomaly is built up due to Ekman divergence, resulting in upwelling over the northern side of LB. The corresponding downwelling signal on the southern side of the bay is due to Ekman convergence. The near-surface currents are very weak over northwestern Rose Bay (see Fig. 1) with relatively weak pressure gradients across the bay, since the downwelling Kelvin waves generated on the southern side of LB switch off the upwelling waves generated on the northern side of LB.

![Fig. 4. Model-calculated near-surface (1.5 m) currents and density anomaly at days (a) 235.5, (b) 236.5 and (c) 237.5 produced by the linear multi-mode model in experiment Mode-LB. Open arrows denote wind stress vectors. Velocity vectors (solid arrows) are plotted at every fourth model grid point.](image)

![Fig. 5. Comparisons of eastward components of observed and model-calculated currents at depths of (a) 3.5 m, (b) 6.5 m and (c) 8.5 m at SB2; at depths of (e) 3.5 m, (f) 6.5 m and (g) 9.5 m at SB3. (d) Comparison of the 10°C isotherm depth estimated from observations and the $s_t = \frac{24}{24} \rho_{0}$ isopycnal depth estimated from model results at SB2. (h) Comparison of the 7°C isotherm depth estimated from observations and the model-calculated $s_t = \frac{24}{24} \rho_{0}$ isopycnal depth at SB3. Outcropping of the 10°C isotherm at SB2 is marked by a shaded box in (d). The model results are produced by the linear multi-mode model in experiment Mode-LB. Correlation coefficients $r$ and $\gamma^2$ statistics are included, where $\gamma^2$ is the ratio of the variance of model errors to the variance of observations.](image)
Rose Bay. By day 236.5 the wind stress has decreased by about 50%, and the near-surface currents in LB (Fig. 4b) become very weak and are nearly northwestward. This is a consequence of the upwelling Kelvin waves generated on the northern side of LB that propagate cyclonically around the head of the bay and gradually set up a weak across-bay pressure gradient in an opposite direction in comparison to that on day 235.5. The eastward currents build up on the northern side of Rose Bay at day 236.5 with downwelling waves exiting Rose Bay. When the wind stress is relaxed at day 237.5, Kelvin waves continue to propagate on the southern side of LB, and strengthen the northwestward currents in LB (Fig. 4c). It should be noted that the upwelling and downwelling in Upper South Cove (USC) and Lower South Cove (LSC) (Fig. 1) in this experiment are not realistic due to the unrealistic representation of local bottom topography.

Fig. 5 presents time series of eastward currents and isotherm depths at SB2 and SB3 based on the observations and model results in experiment Mode-LB. The variance, correlation coefficient and $\gamma^2$ statistics are calculated to quantify the model performance (Tables 2 and 3). Here $\gamma^2$ is defined as the ratio of the variance of the difference between the model results and the observations to the variance of the observations. In this study, $\gamma^2 = 1$ is chosen as a threshold value for assessing the model performance, with smaller values of $\gamma^2$ indicating better model performance. The correlation coefficients are generally greater than 0.5 (significantly different from zero at the 5% level), and the $\gamma^2$ values are smaller than unity for the currents below 6 m depth at SB2 and SB3, indicating that some variabilities of observed sub-surface currents are captured by the linear baroclinic dynamics local to LB. For the near-surface currents, the correlation coefficients are small (not statistically significant from zero) and $\gamma^2$ values are greater than unity, suggesting that the linear multi-mode model has unsatisfactory skills in reproducing the near-surface observed currents at the two sites. The multi-mode model also underestimates the variations of observed sub-surface currents (see Table 2 and Fig. 5f for an example), implying other physical processes such as realistic surface boundary conditions and topography and nonlinear advections are needed to capture the observed variances.

The observed temperatures are used to estimate the position of the 7 °C isotherm at SB3 and 10 °C isotherm at SB2. The 7 and 10 °C isotherms are chosen at SB3 and SB2 respectively because they occurred in the vertical range of temperatures during the most time of the study period. The superposition of the background density profile and model-calculated density anomaly is analysed to find the depths of isopycnal surfaces of $\sigma_i = 24.98 \text{ kg m}^{-3}$ at SB3 and $\sigma_i = 24.68 \text{ kg m}^{-3}$ at SB2, since the initial positions of these two isopycnals are roughly at the same depths of the isotherms. The linear multi-mode model has certain skills in reproducing the variability of the observed isotherm movements at the two sites (Figs. 5d and h), with the correlation coefficients of $\sim 0.54$ at SB2 and $\sim 0.64$ at SB3 (significantly different from zero at the 5% level). In particular, the upwelling and downwelling are captured during the period before day 235 and after day 245, associated mainly with the Ekman divergence and convergence induced by the across-bay (northeastward and southwardwest) wind forcing and the propagation of baroclinic Kelvin waves in LB. For example, when the wind starts blowing northeastward, upwelling (Ekman divergence) and baroclinic jets are developed over western LB, and upwelling Kelvin waves generated on the western side of the bay propagate along the southern side of the bay with the coast on the right. There are large discrepancies, however, between the observed and modeled isotherm depths from day 235 to 245 at the two sites. It will be shown in the next section that the model deficiency is mainly associated with propagation of Kelvin waves from MB to LB which is not included in experiment Mode-LB.

### 3.2. Nonlocal effect of MB

MB is included within the model domain in experiment Mode-MLB. Since Kelvin waves travel with the coastline on the right in the northern hemisphere, baroclinic Kelvin waves generated in MB can propagate into LB. The near-surface currents and density anomaly fields in MB differ from those in LB due mainly to the propagation of baroclinic Kelvin waves. At day 235.5 (Fig. 6a), a southwestward Ekman drift develops in central MB where the Coriolis force primarily balances the wind stress and the local acceleration. Baroclinic coastal jets develop on the northern and southern sides of MB, and upwelling (downwelling) is generated on the northern (southern) side of MB, again due to Ekman divergence (convergence). By day 236.5 (Fig. 6b), the Ekman currents over central MB become weaker and flow southeastward due to the decreasing amplitude of the wind stress and the inertial oscillation. The coastal jets gradually progress downstream over northern and southern MB associated with the propagation of baroclinic Kelvin waves. Weak northward baroclinic jets and upwelling gradually develop along the western shore of MB again due mainly to the propagation of baroclinic Kelvin waves. At day 237.5 (Fig. 6c), the

### Table 2

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Fig. 6. Model-calculated near-surface (1.5 m) currents and density anomaly at day (a) 235.5, (b) 236.5 and (c) 237.5 produced by the linear multi-mode model in experiment Mode-MLB. Open arrows denote wind stress vectors. Velocity vectors (solid arrows) are plotted at every fourth model grid point.

Fig. 7. Comparisons of eastward components of observed and model-calculated currents at depths of (a) 3.5 m, (b) 6.5 m and (c) 8.5 m at SB2; at depths of (e) 3.5 m, (f) 6.5 m and (g) 9.5 m at SB3. (d) Comparison of the 10°C isotherm depth estimated from observations and the σ₀ = 24.68 kg m⁻³ isopycnal depth estimated from model results at SB2. (h) Comparison of the 7°C isotherm depth estimated from observations and the model-calculated σ₀ = 24.98 kg m⁻³ isopycnal depth at SB3. Outcropping of the 10°C isotherm at SB2 is marked by a shaded box in (d). The model results are produced by the linear multi-mode model in experiment Mode-MLB. Correlation coefficients r and γ² statistics are included, where γ² is the ratio of the variance of model errors to the variance of observations.
baroclinic jets flow westward over southern MB after upwelling Kelvin waves have passed by. The near-surface circulation is northward on the eastern shore of MB, also indicative of the Kelvin wave propagation.

A comparison of model results shown in Figs. 4 and 6 demonstrates that the near-surface currents and density anomaly fields in LB are strongly influenced by those in MB particularly at day 236.5 and 237.5. The downwelling signals generated over southern MB turn around East Point Island and propagate westward along the northern side of LB, strengthening the northwestward flow and switching off the intense upwelling in LB.

A comparison of model-calculated sub-surface currents in experiment Mode-MLB with observations at SB2 and SB3 (Figs. 7b, c, f and g) shows that the linear multi-mode model in Mode-MLB has similar skills in capturing the variability of the observed sub-surface currents with correlation coefficients of greater than 0.4 (significantly different from zero at the 5% level) as in Mode-LB. This suggests that the linear baroclinic dynamics can explain the main features of the observed sub-surface currents, including the vertical structure and the direction of the flow, indicative of the importance of baroclinic effects. The correlation between the model-calculated near-surface currents in Mode-MLB and observations (Figs. 7a and e) however, is not statistically significant. The possible explanations for the linear model deficiency in simulating the near-surface currents include the lack of a mixed layer in the formulation of the multi-mode model used here, the lack of advection (e.g. vertical excursions of the mixed layer) as well as the assumption that the density anomaly is always zero at the surface.

The model calculated depths of the isopycnals at SB2 and SB3 in experiment Mode-MLB (Figs. 7d and h) show a significant improvement as compared to those in experiment Mode-LB, indicating that the linear multi-mode model has reasonable skills in simulating the propagation of Kelvin waves from MB to LB. In particular the upwelling/downwelling events between day 235 and 245 are captured by the multi-mode model in Mode-MLB but not in Mode-LB. The correlation coefficients between the observed and calculated isotherm depths produced by the linear model in Mode-MLB are 0.58 at SB2 and 0.79 at SB3 (significantly different from zero at the 5% level, see Table 3).

Fig. 8 demonstrates the relative contribution of the first two baroclinic modes to the variability of the total currents and density anomaly fields produced by the multi-mode model. The total currents at about 3 and 9 m are mainly determined by the first baroclinic mode (Figs. 8c, d and h). The total currents at about 6 m depth (Figs. 8b and f), however, are mainly determined by the second mode, a consequence of the vertical structure of the normal modes (Fig. 3c). The isotherm movements at the two sites are mainly contributed by the first baroclinic mode.

The time-mean currents on the transect running due north from Ovens Point (OP, Fig. 1b) are calculated during the period from day 224 to 249 in experiment Mode-MLB (Fig. 9a). There is generally outflow in the top 4 m and inflow at depth on the transect, and the outflow and inflow are intensified over the northern side of LB. The time-mean currents for the first two baroclinic modes are shown in Figs. 9b and c and demonstrate different horizontal and vertical structures, including the zero-crossing depth and across-bay asymmetry, showing the importance of the higher modes for determining the time-mean model response.

4. The effects of topography and nonlinearity

In this section, we use the fully nonlinear coastal circulation modelling system developed by Zhai et al. (2008) to examine the influence of advection and bottom topography on the baroclinic circulation in LB and MB. The nonlinear coastal circulation modelling system is based on the free surface version of CANDIE (Lu et al., 2001; Sheng and Wang, 2004; Wang et al., 2007), which is a primitive equation ocean circulation model formulated in the
finite-difference form with a z-level coordinate in the vertical. The model uses the hydrostatic and Boussinesq approximations and the fourth order numerics and flux limiter. The details of the model set-up and parameters can be found in Zhai et al. (2008).

Three numerical experiments (Flat-BC, Topo-BC and Topo-BT, see Table 1) are conducted in the domain of LB and MB using the fully nonlinear coastal circulation model. The horizontal resolution in the three experiments is about 200 m. The vertical resolution is 1 m, except for 3 m for the top z-level, and 4.5 m below 20 m depth (where possible). The vertical and horizontal mixing schemes are based on the sub-grid scale mixing parameterization of KPP (K-profile parameterization, Large et al., 1994) and Smagorinsky (1963). A quadratic bottom stress parameterization is used with a drag coefficient of $3 \times 10^{-3}$, apart from in the neighbourhood of USC and LSC where a spatially variable and relatively larger drag coefficient is used (see Sheng and Wang, 2004, for details). The radiation condition of Davies and Flather (1978) is used for the normal velocity along the model open boundaries. The model temperature and salinity at the model open boundaries are weakly restored with a time scale of 10 days to observations taken at Station 2 on the inner Scotian Shelf (see Zhai et al., 2008, for details). In experiments Flat-BC and Topo-BC, the model is initialized from a state of rest with horizontally uniform, but vertically stratified temperature and salinity measured on day 248, 2003, and forced by the observed wind stress (Fig. 2a) and buoyancy fluxes at the sea surface.

![Figure 9](image_url) **Fig. 9.** Vertical distributions of eastward components of model-calculated time-mean currents along the transect due north from Ovens Point produced by the linear multimode model in experiment Mode-MLB. (a) Total solution, (b) solution of the first baroclinic mode and (c) solution of the second baroclinic mode.

![Figure 10](image_url) **Fig. 10.** Model-calculated near-surface currents and temperatures (1.5 m) at day (a) 235.5, (b) 236.5 and (c) 237.5. The model results are produced by the nonlinear coastal circulation model in the domain of LB and MB with realistic topography (Topo-BC). Open arrows denote wind stress vectors. Velocity vectors (solid arrows) are plotted at every fourth model grid point.
computed from observations by Zhai et al. (2007). In experiment Topo-BT, the model is run in barotropic mode with the model temperature and salinity set to be invariant in time and space, and forced by the observed wind stress only. A flat bottom of 15 m is used in experiment Flat-BC. The realistic bottom topography of LB and MB is used in experiments Topo-BC and Topo-BT. The nonlinear model is integrated for 25 days from August 13 to September 7, 2003 in the three numerical experiments.

The near-surface currents and temperatures produced by the nonlinear coastal circulation model in experiment Topo-BC are shown in Fig. 10. At day 235.5, the near-surface currents (Fig. 10a) flow eastward on the northern side and southeastward on the southern side of LB. There is a strong southward flow outside LB. By day 236.5 when the wind forcing becomes weak (Fig. 10b), a small-scale cyclonic gyre is formed over eastern LB as a result of the advection and propagation of internal Kelvin waves. The currents over western LB is relatively weak due mainly to the shallow bottom topography. At day 237.5 (Fig. 10c), the cyclonic gyre gradually disappears, and the currents flow northwestward in LB and are in balance with the across-bay pressure gradient set-up by the internal Kelvin waves. The simulated near-surface temperatures in Topo-BC (Fig. 10) also have strong horizontal

![Fig. 11. Model-calculated near-surface currents and temperatures (1.5 m) at day (a, d) 235.5, (b, e) 236.5 and (c, f) 237.5; (a–c) from the experiment Flat-BC; (d–f) from the experiment Topo-BT. Open arrows denote wind stress vectors. Velocity vectors (solid arrows) are plotted at every fourth model grid point.](image-url)
structures in LB, associated with upwelling and downwelling induced by Ekman dynamics and Kelvin wave propagation. The model-calculated temperature in USC and LSC is relatively warm due to the shallow water depth in the two coves.

A comparison of near-surface currents in experiments Topo-BC (Fig. 10) and Mode-MLB (Fig. 6) shows similar large-scale circulation patterns, including features of the pressure-driven flow near the coast of the two bays and Ekman currents over the interior of MB and outside the two bays. There are significant differences, however, in small-scale circulation features between the two experiments, indicative of the influence of advection and topography. In particular, the formation of the gyre circulation at the entrance of LB is mainly associated with the momentum advection since a model run using a version of CANDIE linearized about a state of rest does not reproduce the gyre (not shown), and the weaker circulation over western LB is associated with the shallow bottom topography.

The large-scale horizontal distributions of the near-surface currents in experiment Topo-BC (Fig. 10) are very similar to those in the flat-bottom case (Flat-BC, Figs. 11a–c). However, the small-scale circulation features in the two experiments differ significantly due to the topographic effect. At day 237.5, the anticyclonic circulation in MB in experiment Topo-BC is restricted by the closed isobaths, whereas there is strong southward flow through the entrance of MB in experiment Flat-BC. A comparison of temperature fields in the two cases demonstrates that the near-surface water in Flat-BC is generally warmer than that in Topo-BC over southern MB and western LB, due to the lack of a cold water source at depths greater than 15 m in the flat-bottom case.

The near-surface barotropic currents shown in Figs. 11d–f are characterized by downwind flow where the barotropic pressure mainly balances the wind stress minus bottom stress, with some contributions from Coriolis and advection terms. The barotropic circulation returns to a state of rest very quickly after the wind dies down due to the large shelf wave speed ($fD$) where $D$ is the width of the slope of the inner Scotian shelf). A comparison of the near-surface currents in experiments Topo-BC (Fig. 10) and Topo-BT (Figs. 11d–f) clearly demonstrates that baroclinic pressure gradients and propagation of baroclinic coastal trapped waves (mainly Kelvin waves) play a dominant role in the large-scale circulation in the region.

Time series of the observed and model-calculated currents at SB2 and SB3 in experiment Topo-BC are shown in Fig. 12. The nonlinear coastal circulation model reproduces reasonably well the temporal variability and vertical structure of observed currents, due mainly to the set-up of internal pressure by the propagation of baroclinic Kelvin waves, and the effects of advection and topography. The correlation coefficients (Table 3) are generally greater than 0.4 (significantly different from zero at the 5% level), and the $\gamma^2$ values are generally smaller than 1 at SB2 and SB3. A comparison of the current variances calculated from model results in each experiment (Table 2) shows that the observed current variance is better captured by the nonlinear coastal circulation model, and is generally underestimated by the linear multi-mode model. Figs. 12d and h show the comparison between the calculated isotherm depths in experiment Topo-BC and the observations at SB2 and SB3. The nonlinear model results show a better agreement with observations as compared to the results in experiment Mode-MLB during the period before day 240, due mainly to the advective and topographic effects. The isotherm is lifted up by about 5–8 m during upwelling, and returns to its original position during downwelling.

![Fig. 12](image_url)

Fig. 12. Comparisons of eastward components of observed and calculated currents (in units of cm s$^{-1}$) at depths of (a) 3.5 m, (b) 6.5 m, and (c) 8.5 m at SB2; at depths of (e) 3.5 m, (f) 6.5 m, and (g) 9.5 m at SB3. (d) Comparison of the 10°C isotherm depth estimated from observations and model results at SB2. (h) Comparison of the 7°C isotherm depth estimated from observations and model results at SB3. Outcropping of the 10°C isotherm at SB2 is marked by a shaded box in (d). The model results are produced by the nonlinear coastal circulation model in the domain of LB and MB with realistic topography (Topo-BC). Correlation coefficients $r$ and $\gamma^2$ statistics are included, where $\gamma^2$ is the ratio of the variance of model errors to the variance of observations.
To further demonstrate the baroclinic effect on the vertical structure of currents, we examine the vertical profiles of daily averaged currents from observations and model results at SB3 (Fig. 13). The vertical profiles of model currents in experiment Topo-BC are in reasonable agreement with the observations, whereas the vertical profiles in Topo-BT are significantly different from the observations, indicative of the importance of baroclinicity. At day 235 when the daily mean wind stress is relatively strong and southeastward, the currents above 5 m flow out of the bay and flow into the bay at depth. By day 236 when the wind stress becomes weaker, the near-surface currents still flow out of the bay, but with reducing magnitudes. The currents at deeper layers gradually reverse directions and start flowing out of the bay, as a consequence of the baroclinic Kelvin wave propagation. While the wind stress almost dies down on day 237, the downwelling Kelvin waves still propagate from southern MB into LB and built up the across-bay baroclinic pressure gradients, intensifying the near-surface inflow and sub-surface outflow.

We also examine the vertical distribution of time-mean currents across the transect due north from OP during the study period from day 224 to day 249. This transect controls the exchange of water mass between the inner shelf and the bay. The similarity of the mean flow across the transect between the cases of Flat-BC (Fig. 14a) and Mode-MLB (Fig. 9a) again suggests that the higher-order baroclinic modes play a very important role in determining the zero crossing of the mean flow. A comparison...
of the time-mean flows in experiments Flat-BC and Topo-BC (Figs. 14a and b) demonstrates that the slope topography at the bay entrance induces a stronger inflow (into the bay) centred at 6–8 m depth on the southern side of LB. The time-mean flow across the transect in the barotropic case (Fig. 14c) differs significantly from that in experiment Topo-BC, again indicating that baroclinicity determines the basic dynamics of the wind-driven circulation in the bay. The time-mean flow in experiment Topo-BT is characterized by the outflow over northern LB driven by along-bay wind stress and inflow over southern LB driven by the along-bay barotropic pressure gradient. This feature is supported by the theoretical and numerical studies in Winant (2004) and Sanay and Valle-Levinson (2005).

5. Summary and discussion

Observations made by a multi-disciplinary ocean observatory in Lunenburg Bay (LB) of Nova Scotia, Canada, suggest that temperatures, salinities and currents have significant temporal variations of several days, which are strongly affected by the wind forcing (Zhai et al., 2007). Five numerical experiments were conducted in this study to examine the propagation of sub-inertial baroclinic Kelvin waves, coastal upwelling and downwelling and topographic and nonlinear effects.

A linear baroclinic dynamical model based on the normal mode approach was first used to examine the baroclinic response to realistic wind forcing in idealized coastal areas which have the same geometry of LB and Mahone Bay (MB) with a flat bottom. The multi-mode model was applied to two domains: one covering LB (Mode-LB) and the other covering LB and MB (Mode-MLB). The model incorporates continuous stratification and is driven by the observed wind. The differences between the model results of the two experiments clearly demonstrate the strong influence of baroclinic Kelvin waves generated on the southern shore of MB on the currents and isotherm movement in LB. The linear multi-mode model shows some skill in simulating the vertical movements of observed isotherms and the variability of the sub-surface currents at the two moorings, indicative of the importance of the baroclinic dynamics. A factor contributing to the discrepancy between the near-surface currents produced by the linear multi-mode model and the observations is likely to be the lack of a mixed layer and the lack of vertical excursions of the mixed layer.

The wind-driven baroclinic circulation and thermal structure are reproduced by a three-dimensional coastal circulation modelling system which includes the effects of nonlinearity and realistic bottom topography. The comparison of model results in experiment Topo-BC and observations shows that the coastal circulation model reproduces reasonably well the variability of observed currents and isotherm depths, including the near-surface currents at the two moorings. Two more numerical experiments have been carried out to illustrate the nonlinear and topographic effects. The nonlinearity is responsible for generating a gyre circulation over eastern LB. The realistic topography significantly affects the vertical structure of time-mean currents across a transect at the mouth of LB.

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Appendix A. The multi-mode model

Readers are referred to McCreary (1981a, b) for a detailed description of the linear baroclinic model based on the normal mode approach. Only a brief discussion is given here. The Boussinesq, rigid-lid, and hydrostatic approximations are made to the model equations. Following McCreary (1981a), the linearized model equations can be separated into vertical normal modes and horizontal mode coefficients in a flat-bottom ocean and possess separable solutions of the form

\[ u = \sum_{n=1}^{N} u_n \psi_n, \quad v = \sum_{n=1}^{N} v_n \psi_n, \quad p = \sum_{n=1}^{N} p_n \psi_n, \]

\[ w = \sum_{n=1}^{N} w_n \Phi_n, \quad \rho / N_e^2 = \sum_{n=1}^{N} \rho_n \Phi_n, \]

where \( u, v, w \) are the velocity components, \( \rho \) is the density anomaly, \( p \) is the internal pressure anomaly, \( N_e \) is the buoyancy frequency, the mode coefficients \( u_n, v_n, w_n, p_n, \rho_n \) are functions only of \( x, y, t, \) and \( \psi_n \) and \( \Phi_n \) are the vertical structure functions. The governing equations for the vertical structure functions are given by,

\[ \frac{\partial}{\partial z} \left( \frac{1}{N_e^2} \frac{\partial \psi_n}{\partial z} \right) + \frac{1}{c_n^2} \psi_n = 0, \]

\[ \frac{\partial^2 \Phi_n}{\partial z^2} + \frac{N_e^2}{c_n^2} \Phi_n = 0, \]

subject to boundary conditions at the sea surface (\( z = 0 \)) and bottom (\( z = -H \)) of,

\[ \Phi_n = 0, \quad \frac{\partial \psi}{\partial z} = 0 \text{ at } z = 0, \]

and

\[ \Phi_n = 0, \quad \frac{\partial \psi}{\partial z} = 0 \text{ at } z = -H. \]

In Eqs. (A.2) and (A.3), \( c_n \) is the baroclinic wave speed for the \( n \)th mode.

The governing equations for the mode coefficients have the same form as the shallow water equations:

\[ \frac{\partial u_n}{\partial t} + \frac{1}{\rho_0} \frac{\partial p_n}{\partial x} + F_n - A \frac{\partial u_n}{\partial y} = 0, \]

\[ \frac{\partial v_n}{\partial t} + \frac{1}{\rho_0} \frac{\partial p_n}{\partial y} + G_n - A \frac{\partial v_n}{\partial x} = 0, \]

\[ \frac{\partial w_n}{\partial t} + \frac{\partial u_n}{\partial x} + \frac{\partial v_n}{\partial y} = 0, \]

\[ \frac{\partial p_n}{\partial t} + \rho_0 g w_n = - A \frac{\sigma c_n^2}{\rho_0} p_n, \]

where \( \rho_0 \) is a reference density, \( F_n \) and \( G_n \) are the projections of the eastward and northward components of wind stress on each mode and are given by

\[ F_n = \tau_x/\rho_0 \int_{-H}^{0} \psi_n^2 \, dz, \quad G_n = \tau_y/\rho_0 \int_{-H}^{0} \psi_n^2 \, dz. \]

Equations (A.6)–(A.10) are solved on an Arakawa C grid using the method given by Heaps (1971). We apply the forward time differencing to the momentum equations and the backward time differencing to the continuity equation. Coriolis terms are computed using the standard four-point averaging.
References


